Schutz 6.38. Integrate $l = \int_{\lambda_0}^{\lambda_1} |\vec{v}\cdot\vec{v}|^{1/2} d\lambda$ to find length of circle of constant θ on sphere of radius r.

$$\vec{v}\cdot\vec{v} = g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}, \qquad \lambda = \phi,$$

$$x^\alpha = \{r, \theta, \phi\}.$$

$$g_{\alpha\beta} = \begin{bmatrix} 1 & & \\ & r^2 & \\ & & r^2\sin^2\theta \end{bmatrix}, \qquad \frac{dx^\alpha}{d\phi} = \{0, 0, 1\}.$$

$$\Rightarrow \vec{v}\cdot\vec{v} = r^2\sin^2\theta$$

$$l = \int_0^{2\pi} |r^2\sin^2\theta|^{1/2} d\phi$$

$$= \int_0^{2\pi} r\sin\theta \, d\phi = \boxed{2\pi r\sin\theta}$$

Davidson Chang

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